

# ON A LINEAR NON-HOMOGENEOUS ORDINARY DIFFERENTIAL EQUATION OF THE HIGHER ORDER WHOSE COEFFICIENTS ARE REAL-VALUED SIMPLE STEP FUNCTIONS

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## Abstract

By using the method developed in the paper [G.Pantsulaia, G.Giorgadze, On some applications of infinite-dimensional cellular matrices, *Georg. Inter. J. Sci. Tech., Nova Science Publishers*, Volume 3, Issue 1 (2011), 107-129], it is obtained a representation in an explicit form of the particular solution of the linear non-homogeneous ordinary differential equation of the higher order whose coefficients are real-valued simple functions.

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## 1. Introduction

In [4] has been obtained a representation in an explicit form of the particular solution of the linear non-homogeneous ordinary differential equation of the higher order with real-valued coefficients. The aim of the present manuscript is resolve an analogous problem for a linear non-homogeneous ordinary differential equation of the higher order when coefficients are real-valued simple step functions.

The paper is organized as follows.

In Section 2, we consider some auxiliary results obtained in the paper [4]. In Section 3, it is obtained a representation in an explicit form of the particular solution of the linear non-homogeneous ordinary differential equation of the higher order whose coefficients are real-valued simple functions. In Section 4 we present mathematical program in MathLab for the graphical solution of the corresponding differential equation.

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## 2. Some auxiliary propositions

For  $n \in \mathbb{N}$ , we denote by  $FD^{(n)}[-l, l[$  a vector space of all  $n$ -times differentiable functions  $\Psi$  on  $[-l, l[$  such that a series obtained by  $k$ -times differentiation term by term of the Fourier trigonometric series of  $\Psi$  pointwise converges to  $\Psi^{(k)}$  for all  $x \in [-l, l[$  and  $0 \leq k \leq n$ .

Let  $(A_n)_{0 \leq n \leq 2M}$  be a sequence of real numbers, where  $M$  is any natural number. For each  $k \geq 1$  we put

$$\sigma_k = \sum_{n=0}^m (-1)^n A_{2n} \left(\frac{k\pi}{l}\right)^{2n}, \quad (2.1)$$

$$\omega_k = \sum_{n=0}^{m-1} (-1)^n A_{2n+1} \left(\frac{k\pi}{l}\right)^{2n+1}. \quad (2.2)$$

**Theorem 2.1.** ([4], Theorem 3.1, p.45) For  $m \geq 1$ , let us consider an ordinary differential equation

$$\sum_{n=0}^{2m} A_n \frac{d^n}{dx^n} \Psi = f, \quad (2.3)$$

where

$$f(x) = \frac{c_0}{2} + \sum_{k=1}^{\infty} c_k \cos\left(\frac{k\pi x}{l}\right) + d_k \sin\left(\frac{k\pi x}{l}\right) \in FD^{(0)}[-l, l[ \quad (2.4)$$

and  $A_n \in \mathbb{R}$  for  $0 \leq n \leq 2m$ .

Suppose that  $A_0 \neq 0$  and  $\sigma_k^2 + \omega_k^2 \neq 0$  for  $k \geq 1$ , where  $\sigma_k$  and  $\omega_k$  are defined by (2.1) and (2.2), respectively.

If  $(\frac{c_0}{2}, c_1, d_1, c_2, d_2, \dots)$  is such a sequence of real numbers that the series  $\Psi_p$ , defined by

$$\Psi_p(x) = \frac{c_0}{2A_0} + \sum_{k=1}^{\infty} \left( \frac{c_k \sigma_k - d_k \omega_k}{\sigma_k^2 + \omega_k^2} \right) \cos\left(\frac{k\pi x}{l}\right) + \left( \frac{c_k \omega_k + d_k \sigma_k}{\sigma_k^2 + \omega_k^2} \right) \sin\left(\frac{k\pi x}{l}\right), \quad (2.5)$$

belongs to the class  $FD^{(2m)}[-l, l[$ , then  $\Psi_p$  is a particular solution of (2.3).

**Theorem 2.2.** ([4], Theorem 3.2, p.45) For  $m \geq 1$ , let us consider an ordinary differential equation (2.3), where

$$f(x) \in C[-l, l] \quad (2.6)$$

and  $A_n \in \mathbb{R}$  for  $0 \leq n \leq 2m$ .

Suppose that  $A_0 \neq 0$  and  $\sigma_k^2 + \omega_k^2 \neq 0$  for  $k \geq 1$ , where  $\sigma_k$  and  $\omega_k$  are defined by (2.1) and (2.2), respectively. Let  $(\frac{c_0}{2}, c_1, d_1, c_2, d_2, \dots)$  be Fourier coefficients of  $f$  and  $(\frac{c_0}{2}, c_1, d_1, c_2, d_2, \dots) \in \ell_1$ .

Then the series  $\Psi_p$ , defined by

$$\Psi_p(x) = \frac{c_0}{2A_0} + \sum_{k=1}^{\infty} \left( \frac{c_k \sigma_k - d_k \omega_k}{\sigma_k^2 + \omega_k^2} \right) \cos\left(\frac{k\pi x}{l}\right) + \left( \frac{c_k \omega_k + d_k \sigma_k}{\sigma_k^2 + \omega_k^2} \right) \sin\left(\frac{k\pi x}{l}\right), \quad (2.7)$$

is a particular solution of (2.3).

### 3. A non-homogeneous ordinary differential equation of higher order whose coefficients are continuous or real-valued step functions

Let consider a partition of  $[-l, l[$  defined by

$$[-l, l[ = \cup_{s=0}^{S-1} \left[ \frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S} \right[$$

We define a differential operator

$$L(\Psi) = \sum_{n=0}^{2m} A_n(x) \frac{d^n}{dx^n} \Psi$$

for  $\Psi \in FD^{(2m)}[-l, l[$ . Notice that  $L(\Psi)$  can be rewritten as follows

$$L(\Psi) = \cup_{s=0}^{S-1} Ind_{\left[ \frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S} \right[} \left( \sum_{n=0}^{2m} A_n(x) \frac{d^n}{dx^n} \right) \Psi$$

for  $\Psi \in FD^{(2m)}[-l, l[$ , where  $Ind$  denotes an indicator function.

For each  $S \in \mathbb{N}$  we define an operator  $L_S$  by

$$L_S(\Psi) = \sum_{s=0}^{S-1} Ind_{\left[ \frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S} \right[} \left( \sum_{n=0}^{2m} A_n \left( \frac{l(2s+1-S)}{S} \right) \frac{d^n}{dx^n} \right) \Psi$$

for  $\Psi \in FD^{(2m)}[-l, l[$ .

**Lemma 3.1.** For each  $\Psi \in FD^{(2m)}[-l, l[$  we have

$$L(\Psi) = \lim_{S \rightarrow \infty} L_S(\Psi).$$

**Theorem 3.2.** For  $m \geq 1$ , let us consider an ordinary differential equation

$$\sum_{n=0}^{2m} A_n(x) \frac{d^n}{dx^n} \Psi = f, \quad (3.1)$$

where

$$f(x) = \frac{c_0}{2} + \sum_{k=1}^{\infty} c_k \cos\left(\frac{k\pi x}{l}\right) + d_k \sin\left(\frac{k\pi x}{l}\right) \in FD^{(0)}[-l, l[ \quad (3.2)$$

and  $A_n(x) \in C[-l, l]$  for  $0 \leq n \leq 2m$ .

Suppose that  $A_0(x) = 1$  and  $\sigma_k^2(x) + \omega_k^2(x) \neq 0$  for  $x \in [-l, l[$  and  $k \geq 1$ , where  $\sigma_k(x)$  and  $\omega_k(x)$  are defined by

$$\sigma_k(x) = \sum_{n=0}^m (-1)^n A_{2n}(x) \left(\frac{k\pi}{l}\right)^{2n}, \quad (3.3)$$

$$\omega_k(x) = \sum_{n=0}^{m-1} (-1)^n A_{2n+1}(x) \left(\frac{k\pi}{l}\right)^{2n+1}. \quad (3.4)$$

Suppose that the following conditions are valid:

(i)  $(\frac{c_0}{2}, c_1, d_1, c_2, d_2, \dots) \in \ell_1$ ;

(iii) There is a constant  $C > 0$  such that

$$\left| \frac{\omega_k(x+h)}{\sigma_k^2(x+h) + \omega_k^2(x+h)} - \frac{\omega_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right| \leq C|h|^2$$

and

$$\left| \frac{\sigma_k(x+h)}{\sigma_k^2(x+h) + \omega_k^2(x+h)} - \frac{\sigma_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right| \leq C|h|^2.$$

Then the function  $\Psi_0$ , defined by

$$\begin{aligned} \Psi_0(x) = & \frac{c_0}{2} + \sum_{k=1}^{\infty} \left( \frac{c_k \sigma_k(x) - d_k \omega_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \cos\left(\frac{k\pi x}{l}\right) + \\ & \left( \frac{c_k \omega_k(x) + d_k \sigma_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \sin\left(\frac{k\pi x}{l}\right), \end{aligned} \quad (3.5)$$

is a particular solution of (3.1).

*Proof.* We put

$$\begin{aligned} \Psi_S(x) = & \sum_{s=0}^{S-1} \text{Ind}_{[\frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S}]}(x) \left[ \frac{c_0}{2} + \right. \\ & \sum_{k=1}^{\infty} \left( \frac{c_k \sigma_k(\frac{l(2s+1-S)}{S}) - d_k \omega_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} \right) \cos\left(\frac{k\pi x}{l}\right) + \\ & \left. \left( \frac{c_k \omega_k(\frac{l(2s+1-S)}{S}) + d_k \sigma_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} \right) \sin\left(\frac{k\pi x}{l}\right) \right], \end{aligned} \quad (3.6)$$

On the one hand, by using the result of Lemma 3.1 we have

$$L(\lim_{S \rightarrow \infty} \Psi_S(x)) = \lim_{S \rightarrow \infty} L(\Psi_S(x)) = \lim_{S \rightarrow \infty} L_S(\Psi_S(x)) = \lim_{S \rightarrow \infty} f(x) = f(x).$$

On the other hand we have

$$\begin{aligned} |\lim_{S \rightarrow \infty} \Psi_S(x) - \Psi_0(x)| = & \lim_{S \rightarrow \infty} |\Psi_S(x) - \Psi_0(x)| = \lim_{S \rightarrow \infty} \left| \sum_{s=0}^{S-1} \text{Ind}_{[\frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S}]}(x) \left[ \left( \frac{c_0}{2} - \frac{c_0}{2} \right) + \right. \right. \\ & \sum_{k=1}^{\infty} \left( \frac{c_k \sigma_k(\frac{l(2s+1-S)}{S}) - d_k \omega_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{c_k \sigma_k(x) - d_k \omega_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \cos\left(\frac{k\pi x}{l}\right) + \\ & \left. \left( \frac{c_k \omega_k(\frac{l(2s+1-S)}{S}) + d_k \sigma_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{c_k \omega_k(x) + d_k \sigma_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \sin\left(\frac{k\pi x}{l}\right) \right] \right| \leq \end{aligned}$$

$$\begin{aligned}
& \lim_{S \rightarrow \infty} \sum_{s=0}^{S-1} \sup_{x \in [\frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S}]} \left\{ \sum_{k=1}^{\infty} \left| \frac{c_k \sigma_k(\frac{l(2s+1-S)}{S}) - d_k \omega_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{c_k \sigma_k(x) - d_k \omega_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right| + \right. \\
& \quad \left| \frac{c_k \omega_k(\frac{l(2s+1-S)}{S}) + d_k \sigma_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{c_k \omega_k(x) + d_k \sigma_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right| \Big\} = \\
& \lim_{S \rightarrow \infty} \sum_{s=0}^{S-1} \sup_{x \in [\frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S}]} \left\{ \sum_{k=1}^{\infty} \left| c_k \left( \frac{\sigma_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{\sigma_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) - \right. \right. \\
& \quad \left. d_k \left( \frac{\omega_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{\omega_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \right| + \\
& \quad \left| d_k \left( \frac{\sigma_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{\sigma_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) + \right. \\
& \quad \left. c_k \left( \frac{\omega_k(\frac{l(2s+1-S)}{S})}{\sigma_k^2(\frac{l(2s+1-S)}{S}) + \omega_k^2(\frac{l(2s+1-S)}{S})} - \frac{\omega_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \right| \Big\} \leq \\
& \lim_{S \rightarrow \infty} \sum_{s=0}^{S-1} \sup_{x \in [\frac{l(2s-S)}{S}, \frac{l(2s+2-S)}{S}]} \left\{ \sum_{k=1}^{\infty} 2(|c_k| + |d_k|) \frac{4l^2 C}{S^2} \right\} \leq \\
& \lim_{S \rightarrow \infty} \frac{8l^2 C}{S} \sum_{k=1}^{\infty} (|c_k| + |d_k|) = 0.
\end{aligned}$$

□

*Remark 3.3.* Theorem 3.2 is a generalization of Theorem 2.2. Indeed, Theorem 2.2 is a simple consequence of Theorem 3.2, when  $A_n(x) = \text{const}$  for  $0 \leq n \leq 2m$ , because in that cases all conditions of Theorem 3.2 are fulfilled.

We say that  $(a_k)_{0 \leq k \leq s}$  is partition of  $[-l, l[$  if  $-l = a_0 < a_1 < \dots < a_{s-1} < a_s = l$ .

We say that a real-valued function  $f$  on  $[-l, l[$  is simple function if there exists a partition  $(a_k)_{0 \leq k \leq s}$  of  $[-l, l[$  and a sequence of real numbers  $(A_k)_{1 \leq k \leq s}$  such that

$$f(x) = \sum_{k=1}^s A_k \text{Ind}_{[a_{k-1}, a_k)}(x)$$

for  $x \in [-l, l[$ .

We have the following proposition.

**Theorem 3.4.** Suppose that  $(A_n(x))_{0 \leq n \leq 2m}$  is a sequence of real-valued simple step functions on  $[-l, l[$ , i.e. for every  $n$  ( $0 \leq n \leq 2m$ ) there exists a partition  $(a_k^{(n)})_{0 \leq k \leq s_n}$  of  $[-l, l[$  and a sequence of real numbers  $(A_k^{(n)})_{1 \leq k \leq s_n}$  such that

$$A_n(x) = \sum_{k=1}^{s_n} A_k^{(n)} \text{Ind}_{[a_{k-1}^{(n)}, a_k^{(n)})}(x)$$

for  $x \in [-l, l[$ .

Suppose that  $A_0(x)$  does not remain a zero value on  $[-l, l[$  and  $\sigma_k^2(x) + \omega_k^2(x) \neq 0$  for  $x \in [-l, l[$  and  $k \geq 1$ , where  $\sigma_k(x)$  and  $\omega_k(x)$  are defined by (3.3) and (3.4). Suppose also that Fourier coefficients of the function  $f$  standing in the right side of the equation (3.1) satisfy the following condition  $(\frac{c_0}{2}, c_1, d_1, c_2, d_2, \dots) \in \ell_1$ .

Then the function  $\Psi_0$ , defined by

$$\begin{aligned} \Psi_0(x) = & \frac{c_0}{2A_0(x)} + \sum_{k=1}^{\infty} \left( \frac{c_k \sigma_k(x) - d_k \omega_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \cos\left(\frac{k\pi x}{l}\right) + \\ & \left( \frac{c_k \omega_k(x) + d_k \sigma_k(x)}{\sigma_k^2(x) + \omega_k^2(x)} \right) \sin\left(\frac{k\pi x}{l}\right), \end{aligned} \quad (3.7)$$

for  $x \in [-l, l[$ , satisfies (3.1)–(3.2) at each point of the set

$$(-l, l) \setminus \bigcup_{0 \leq n \leq 2m} \{a_1^{(n)}, a_2^{(n)}, \dots, a_{s_n-1}^{(n)}\}$$

.

*Proof.* If  $x_0 \in (-l, l) \setminus G$  ( $G := \bigcup_{0 \leq n \leq 2m} \{a_1^{(n)}, a_2^{(n)}, \dots, a_{s_n-1}^{(n)}\}$ ), then by virtue of the openness of the  $G$  there exists a positive real number  $r > 0$  such that  $(x_0 - r, x_0 + r) \subseteq G$ . It is obvious that  $A_n(x)$  is constant on  $(x_0 - r, x_0 + r)$  for  $0 \leq n \leq 2m$ . We set  $A_n := A_n(x_0)$  for  $0 \leq n \leq 2m$ .

For  $m \geq 1$ , let us consider an ordinary differential equation

$$\sum_{n=0}^{2m} A_n \frac{d^n}{dx^n} \Psi = f. \quad (3.8)$$

Note that for (3.8) all conditions of Theorem 2.2 are fulfilled. Hence the series  $\Psi_p$ , defined by

$$\begin{aligned} \Psi_p(x) = & \frac{c_0}{2A_0} + \sum_{k=1}^{\infty} \left( \frac{c_k \sigma_k - d_k \omega_k}{\sigma_k^2 + \omega_k^2} \right) \cos\left(\frac{k\pi x}{l}\right) + \\ & \left( \frac{c_k \omega_k + d_k \sigma_k}{\sigma_k^2 + \omega_k^2} \right) \sin\left(\frac{k\pi x}{l}\right), \end{aligned} \quad (3.9)$$

is a particular solution of (3.8), where  $\sigma_k$  and  $\omega_k$  are defined by (2.1) and (2.2), respectively.

Notice that  $\Psi_p$  defined by (3.9) coincides with  $\psi_0$  defined by (3.7) at all point  $x \in (x_0 - r, x_0 + r)$ . Similarly, the equation (3.8) with (3.2) coincides with the equation (3.1) with (3.2) at all point  $x \in (x_0 - r, x_0 + r)$ . Hence  $\psi_0$  defined by (3.7) satisfies (3.1)–(3.2) at each point of the set  $(x_0 - r, x_0 + r)$ , in particular, at point  $x_0$ . Since  $x_0 \in (-l, l) \setminus G$  was taken arbitrary, we end the proof of Theorem 3.4.  $\square$

#### 4. On a graphical solution of the linear non-homogeneous ordinary differential equation of the higher order whose coefficients are real-valued simple step functions

Let consider the linear non-homogeneous ordinary differential equation of the 22-th order

$$\Psi(x) + A_2(x) \frac{d^2}{dx^2} \Psi(x) + A_5(x) \frac{d^5}{dx^5} \Psi(x) + A_{14}(x) \frac{d^{14}}{dx^{14}} \Psi(x) + A_{20}(x) \frac{d^{20}}{dx^{20}} \Psi(x) +$$

$$A_{22}(x) \frac{d^{22}}{dx^{22}} \Psi(x) = 1 + 2 \cos(x), \quad (4.1)$$

where

$$A_2(x) = -0.001 \times \text{Ind}_{[-\pi, -\pi/2[}(x) - 0.002 \times \text{Ind}_{[-\pi/2, 0[}(x) - 0.001 \times \text{Ind}_{[0, \pi/2[}(x) - 0.002 \times \text{Ind}_{[\pi/2, \pi[}(x),$$

$$A_5(x) = 0.01 \times \text{Ind}_{[-\pi, -\pi/2[}(x) - 0.01 \times \text{Ind}_{[-\pi/2, 0[}(x) + 0.002 \times \text{Ind}_{[0, \pi/2[}(x) - 0.002 \times \text{Ind}_{[\pi/2, \pi[}(x),$$

$$A_{14}(x) = 0.1 \times \text{Ind}_{[-\pi, -\pi/2[}(x) - 0.1 \times \text{Ind}_{[-\pi/2, 0[}(x) - 0.4 \times \text{Ind}_{[0, \pi/2[}(x) + 0.007 \times \text{Ind}_{[\pi/2, \pi[}(x),$$

$$A_{20}(x) = -0.01 \times \text{Ind}_{[-\pi, -\pi/2[}(x) + 0.01 \times \text{Ind}_{[-\pi/2, 0[}(x) + 0.002 \times \text{Ind}_{[0, \pi/2[}(x) - 0.22 \times \text{Ind}_{[\pi/2, \pi[}(x),$$

$$A_{22}(x) = 0.001 \times \text{Ind}_{[-\pi, -\pi/2[}(x) - 0.001 \times \text{Ind}_{[-\pi/2, 0[}(x) + 0.0003 \times \text{Ind}_{[0, \pi/2[}(x) - 0.0003 \times \text{Ind}_{[\pi/2, \pi[}(x).$$

**Definition 4.1** We say that  $g \in FD^{(22)}([- \pi, \pi] \setminus G)$  ( $G := \{-\pi, -\pi/2, 0, \pi/2, \pi\}$ ) if

$$g(x) = g_1(x) \times \text{Ind}_{[-\pi, -\pi/2[}(x) + g_2(x) \times \text{Ind}_{[-\pi/2, 0[}(x) + g_3(x) \times \text{Ind}_{[0, \pi/2[}(x) + g_4(x) \times \text{Ind}_{[\pi/2, \pi[}(x) \quad (4.2)$$

for some  $g_1, g_2, g_3, g_4 \in FD^{(22)}([- \pi, \pi])$ .

Below we present the program in MathLab which gives the graphical solution of the differential equation (4.1) in the class  $FD^{(22)}([- \pi, \pi] \setminus G)$ .

```

A1 = [0, -0.001, 0, 0, 0.01, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0, 0, 0, 0, 0, -0.01, 0, 0.001];
A2 = [0, -0.002, 0, 0, -0.01, 0, 0, 0, 0, 0, 0, 0, 0, -0.1, 0, 0, 0, 0, 0, 0.01, 0, -0.001];
A3 = [0, -0.001, 0, 0, 0.002, 0, 0, 0, 0, 0, 0, 0, 0, -0.4, 0, 0, 0, 0, 0, 0.002, 0, 0.0003];
A4 = [0, -0.002, 0, 0, -0.002, 0, 0, 0, 0, 0, 0, 0, 0, 0.007, 0, 0, 0, 0, 0, -0.22, 0, -0.0003];
C = [2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];
D = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];
C0 = 2; A10 = 1; A20 = 1; A30 = 1; A40 = 1;
x = 1 : 20;
S1 = A10; S2 = A20; S3 = A30; S4 = A40;
for k = 1 : 11
    S1 = S1 + (-1)(k) * A1(2*k) * x.(2*k);
    S2 = S2 + (-1)(k) * A2(2*k) * x.(2*k);
    S3 = S3 + (-1)(k) * A3(2*k) * x.(2*k);
    S4 = S4 + (-1)(k) * A4(2*k) * x.(2*k);
end
O1 = A1(1); O2 = A2(1); O3 = A3(1); O4 = A4(1);
for k = 1 : 10

```

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O1 = O1 + (-1)^k * A1(2*k+1) * x.(2*k+1);
O2 = O2 + (-1)^k * A2(2*k+1) * x.(2*k+1);
O3 = O3 + (-1)^k * A3(2*k+1) * x.(2*k+1);
O4 = O4 + (-1)^k * A4(2*k+1) * x.(2*k+1);
end
x1 = (-pi) : (pi/100) : (-pi/2);
y1 = C0/(2*A10);
for n = 1 : 20
y1 = y1 + cos(n*x1) .* (C(n)*S1(n) - D(n)*O1(n))/(S1(n)^2 + O1(n)^2) +
sin(n*x1) .* (C(n)*O1(n) + D(n)*S1(n))/(S1(n)^2 + O1(n)^2);
end
x2 = (-pi/2) : (pi/100) : 0;
y2 = C0/(2*A20);
for n = 1 : 20
y2 = y2 + cos(n*x2) .* (C(n)*S2(n) - D(n)*O2(n))/(S2(n)^2 + O2(n)^2) +
sin(n*x2) .* (C(n)*O2(n) + D(n)*S2(n))/(S2(n)^2 + O2(n)^2);
end
x3 = 0 : (pi/100) : (pi/2);
y3 = C0/(2*A30);
for n = 1 : 20
y3 = y3 + cos(n*x3) .* (C(n)*S3(n) - D(n)*O3(n))/(S3(n)^2 + O3(n)^2) +
sin(n*x3) .* (C(n)*O3(n) + D(n)*S3(n))/(S3(n)^2 + O3(n)^2);
end
x4 = (pi/2) : (pi/100) : pi;
y4 = C0/(2*A40);
for n = 1 : 20
y4 = y4 + cos(n*x4) .* (C(n)*S4(n) - D(n)*O4(n))/(S4(n)^2 + O4(n)^2) +
sin(n*x4) .* (C(n)*O4(n) + D(n)*S4(n))/(S4(n)^2 + O4(n)^2);
end
for i = 1 : 20
if O1(i)^2 + S1(i)^2 == 0; O2(i)^2 + S2(i)^2 == 0; O3(i)^2 + S3(i)^2 == 0; O3(i)^2 + S3(i)^2 ==
0;
plot(x1,y1,x2,y2,x3,y3,x4,y4)
else error('the ordinary differential equation has no solution or has infinitely many
solutions' in the class FD^(22)([-pi,pi] \ G))
end
end
end

```

On Figure 1, the graphical solution of the differential equation (4.1) is presented.

**Remark 4.1** Notice that for each natural number  $M > 1$ , one can easily modify this program in MathLab for obtaining a graphical solution of the differential equation (3.1)-(3.2) in  $FD^{(2M)}([-l, l] \setminus G)$  whose coefficients  $(A_n(x))_{0 \leq n \leq 2M}$  are real-valued simple step functions on  $[-l, l]$ ,  $f$  is a trigonometric polynomial on  $[-l, l]$  and  $G$  is the partition of the interval  $[-l, l]$  defined by the family  $(A_n(x))_{0 \leq n \leq 2M}$ .



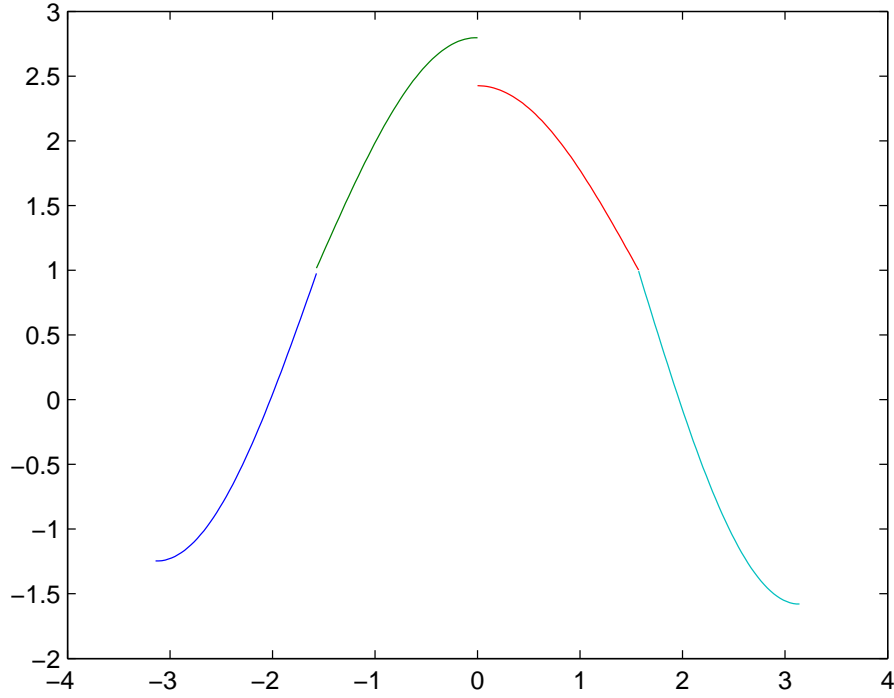


Figure 1. Graphical solution of the ODE (4.1).

**Remark 4.2** Since each constant  $c$  admits the following evident representation

$$c = c \times \text{Ind}_{[-\pi, -\pi/2[}(x) + c \times \text{Ind}_{[-\pi/2, 0[}(x) + c \times \text{Ind}_{[0, \pi/2[}(x) + c \times \text{Ind}_{[\pi/2, \pi[}(x), \quad (4.3)$$

we can use above mentioned program for a solution of the differential equation (2.3)-(2.4) with constant coefficients.

On Figure 2, the graphical solution of the linear non-homogeneous ordinary differential equation of the second order with real-valued constant coefficients

$$\Psi(x) - \frac{d^2}{dx^2}\Psi(x) = 1/2 + \cos(x), \quad (4.4)$$

is presented, which has been obtained by entering in the above mentioned program of the following data:

$A1 = [0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];$   
 $A2 = [0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];$   
 $A3 = [0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];$   
 $A4 = [0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];$   
 $C = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];$   
 $D = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];$   
 $C0 = 1; A10 = 1; A20 = 1; A30 = 1; A40 = 1;$

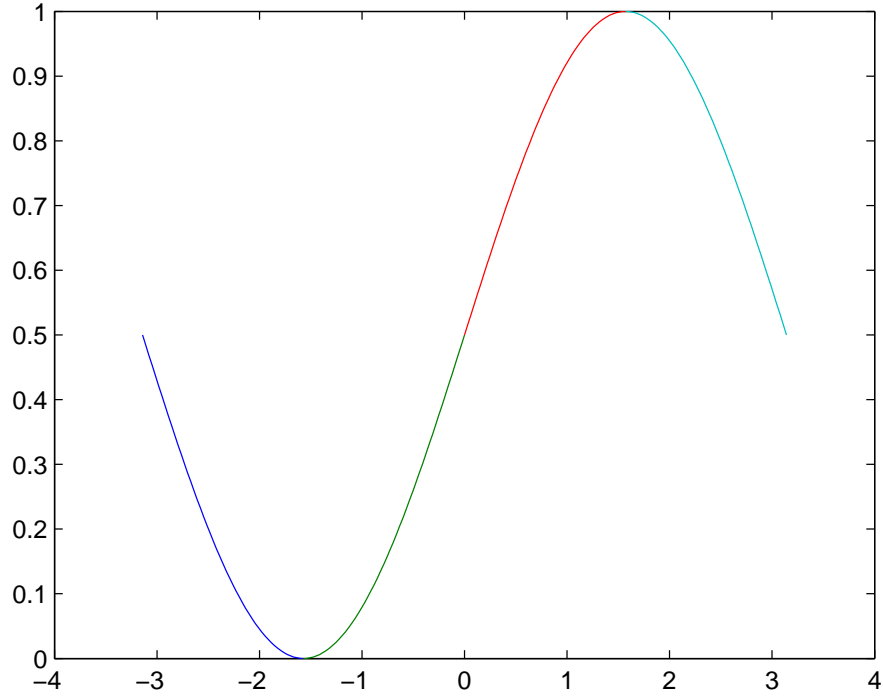


Figure 2. Graphical solution of the ODE (4.4).

*Remark 4.1.* The approach of Theorem 3.4 used for a solution of (3.1)-(3.2) with real-valued simple step functions  $(A_n(x))_{0 \leq n \leq 2M}$  can be used in such a case when the corresponding coefficients are continuous functions on  $[-l, l[$ . If we will approximate these coefficients by real-valued simple step functions, then it is natural to wait that under some "nice restrictions" on these coefficients the solution obtained by Theorem 3.4, will be a "good approximation" of the corresponding solution.

## References

- [1] Linear differential equation, [http://en.wikipedia.org/wiki/Linear\\_differential\\_equation](http://en.wikipedia.org/wiki/Linear_differential_equation).
- [2] G.Birkhoff, G. Rota, *Ordinary Differential Equations*, New York: John Wiley and Sons, Inc., 1978.
- [3] J.C. Robinson, *An Introduction to Ordinary Differential Equations*, Cambridge, UK.: Cambridge University Press, 2004.
- [4] G.Pantsulaia, G.Giorgadze, On some applications of infinite-dimensional cellular matrices, *Georg. Inter. J. Sci. Tech., Nova Science Publishers*, Volume 3, Issue 1 (2011), 107-129.

- [5] A. Stanoyevitch, *Introduction to MATLAB<sup>®</sup> with numerical preliminaries*, Wiley-Interscience [John Wiley & Sons], Hoboken, NJ, 2005. x+331 pp.